Flutter of Orthotropic Panels in Supersonic Flow Using Affine Transformations

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Affine transformations are used in analyzing the flutter problem of rectangular simply supported orthotropic panels subjected to supersonic flow over one surface. With the help of certain defined characteristic and bounded quantities a comprehensive solution, which has the isotropic panels solution as a subset, is found to this problem. The physics of this very important aeroelastic problem which has been so obscure for a long time because of the presence of so many parameters is thus clearly exposed by showing how the aerodynamic and the elastic forces interact to produce the panel flutter phenomenon. Both the aerodynamic strip (Ackeret) theory and lifting surface theory are compared and found to agree very well in the analysis. Hence, complete stability boundaries are determined for both flat and buckled panels using the aerodynamic strip theory, the simpler of the two theories.

Nomenclature

A	$=(k_0-2n^2D^*)(a_0/b_0)^2$
(a,b), (a_0,b_0)	= dimensions of panel in Cartesian and affine planes, respectively
a_{mn}, (q_m,a_m)	= amplitudes of (mn) and (m) mode, respectively
c_∞	= sound velocity
D^*	= generalized rigidity ratio $[=(D_{12}+2D_{66})/(D_{11}D_{22})^{1/2}]$
D_{ij}	= elastic constants
h	= panel thickness
k	$=\rho h a_0^4\omega^2-(\pi a_0 n/b_0)^4+(na_0/b_0)^2\pi^4 k_{y0}$
k_0	$=N_{x0}b_0^2/\pi^2$
k_0^T	$=h\beta_1\alpha_0\Delta T b_0^2/\pi^2$
k_{y0}	$=N_{y0}a_0^2/\pi^2$
k_{cr},λ_{cr}	= critical k and λ, respectively
L,L_0	= lateral aerodynamic loading in Cartesian and affine planes, respectively
$\bar{L}_{mn,rs}$	= generalized force of (rs) mode on (mn) mode
M	= Mach number
m,n,r,s,N,ℓ,i,j	= integers
(N_x,N_y), (N_{x0},N_{y0})	= midplane stresses (+ in compression) in Cartesian and affine planes, respectively
q_0	= dynamic pressure in the affine plane $[=\frac{1}{2}(\rho_\infty U^2)/(D_{11})^{1/4}]$
ΔT	= temperature rise
t	= time
U	= air flow velocity
W	= lateral deflection of panel
W_s, W_d	= static and dynamic panel deflections, respectively
(x,y,z), (x_0,y_0,z)	= Cartesian and affine coordinates, respectively
α	$=(M^2-1)^{1/2}$
α_0	= thermal coefficient
β_1	= support factor
$\delta_0(\eta)$	= step function $(=1,\eta<0,=0,\eta<0)$
ϵ	= generalized Poisson's ratio $\{=D_{12}/[D^*(D_{11}D_{22})^{1/2}]\}$
ϵ_0	= structural damping factor
λ	= dynamic pressure parameter in the affine plane $(=2q_0a_0^3/\alpha)$
ξ	$=x_0/a_0$
ρ	= panel mass density
ρ_∞	= air mass density
Φ	= perturbation potential
ω	= circular frequency
ω_0	= frequency parameter $[=(\rho h a_0^4\omega^2)^{1/2}/\pi^2]$

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Introduction

OVER the years the branch of aeroelasticity concerned with the peculiar phenomenon of panel flutter has benefited from many outstanding contributions. The works of the individuals cited in Refs. 1-18 are just a few examples. As a result, this self-excited oscillation of the external skin of a flight vehicle when exposed to an airflow on one side (panel flutter) is now reasonably understood. However, most of the analyses have been restricted to isotropic panels using highly simplified (but accurate) aerodynamics.

In recent years skyrocketing energy costs have made minimum weight aircraft more necessary than ever. Many people believe composite materials comprising laminates of orthotropic or anisotropic materials would help achieve this goal. Consequently, more serious attention has been given to the study of orthotropic (anisotropic) panels. In the area of aeroelastic instability of these panels, the works of Perlmutter,[13] Calligeros and Dugundji,[14] and Ketter,[15] among others, have been very enlightening. However, the persistent presence of so many parameters (in the equations), whose relative importance and bounds are unknown has made the physics of this very important aeroelastic phenomenon obscure. For instance, most of the analyses either ignored the very crucial effects of the midplane compressive forces parallel to the flow or carry them through the early stages of the calculations and drop them when the stability boundaries are computed. As a result, nothing is said about the buckling effects on the flutter boundaries as is usually the case with the isotropic panels. The reason for this is the lack of availability of buckling (and vibration) data for orthotropic panels (with the exception of a few panels).

This state of affairs, therefore, led the author to the conclusion that in order to do a meaningful orthotropic panel flutter analysis, it would be necessary to carry out a comprehensive buckling and vibration analysis of these panels. Consequently, work was begun by presenting a complete solution and master curves for the buckling and vibration of the entire orthotropic panels in Ref. 19. Having done that, this paper sets out to show that with the help of the affine transformations, a generalized form of solution can be obtained for orthotropic panels from which the solutions presented by Hedgepeth,[2] Dowell and Voss,[3] Houbolt,[6] Bisplinghoff and Ashley,[7] and others for isotropic panels can be recovered. Thus, a unified analysis can be achieved.

The key, as provided by Brunelle,[20,21] is to look for similarity rules for the orthotropic panels using the affine transformations. When this is done, two main advantages of the affine plane become evident. First, the equations have only two characteristic and bounded elastic parameters; the generalized rigidity ratio, varying between zero and one for all panels (isotropic panels have a value of one), and the generalized Poisson's ratio, the range of which is the same as that of isotropic panels. Second, the effects of the midplane stresses resulting from loading and aerodynamic heating are clearly visible. Consequently, both the exact and Galerkin's solutions for flat and buckled panels require only a little more algebra than that of the isotropic panels. Thus, the isotropic and orthotropic panel theories can be combined into a single theory (see also Ref. 22). Aerodynamic strip theory is used for the most part in the analysis while the three-dimensional aerodynamic theory (surface theory) is only used to prove the accuracy of the strip theory.

Statement of Problem

The analysis considers a flat, rectangular orthotropic panel of length a, width b, and a uniform thickness h, shown in Fig. 1. The panel is simply supported at the edges and subjected to a supersonic airflow U over its top surface and midplane force intensities N_x and N_y.

Equations of Motion

Using classical plate assumptions and either the variational principles or the "picture" method, the governing partial differential equations of motion and boundary conditions are

$$D_{11}\frac{\partial^4 W}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 W}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 W}{\partial y^4}+\rho h\frac{\partial^2 W}{\partial t^2}$$

$$+N_x\frac{\partial^2 W}{\partial x^2}+N_y\frac{\partial^2 W}{\partial y^2}+L+\rho h\epsilon_0\frac{\partial W}{\partial t}=0$$

$$W(a,y,t)=\frac{\partial^2 W}{\partial x^2}(0,y,t)=\frac{\partial^2 W}{\partial x^2}(a,y,t)=\frac{\partial^2 W}{\partial y^2}(x,0,t)$$

$$=\frac{\partial^2 W}{\partial y^2}(x,b,t)=0 \tag{1}$$

where L, the airforces per unit area, is assumed to be adequately derivable from the linearized static aerodynamic theory. Thus, the loading at every instant is assumed to be the loading which would result from flow over a static surface with shape equal to that of the deflected plate at that instant. Hence, we can ignore the time-dependent effects in the potential equation, the boundary conditions, and the pressure-potential relations. For simplicity, structural damping will also be ignored in the analysis. A comprehensive analysis of Eq. (1) will now be carried out with the help of affine transformations.

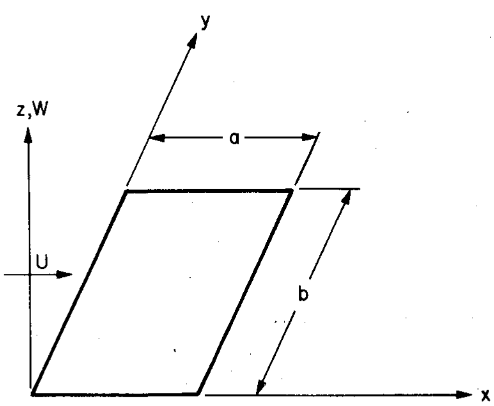

Fig. 1 A rectangular orthotropic panel in Cartesian coordinate system.

Analysis

Affine Transformations

Let $x=(D_{11})^{1/4}x_0$, $y=(D_{22})^{1/4}y_0$

$$D^*\triangleq\frac{(D_{12}+2D_{66})}{(D_{11}D_{22})^{1/2}},\qquad \epsilon\triangleq\frac{D_{12}}{D^*(D_{11}D_{22})^{1/2}}$$

$$N_{x_0}\triangleq\frac{N_x}{(D_{11})^{1/2}},\qquad N_{y_0}\triangleq\frac{N_y}{(D_{22})^{1/2}} \tag{2}$$

where D^* is called the "generalized rigidity ratio" and varies between 0 and 1 for all orthotropic panels and ϵ is the "generalized Poisson's ratio," and varies in a manner similar to that of isotropic panels.

Ackeret's Strip Theory in the Affine Plane

Lateral loading L_0 using Ackeret's theory[23] is given by

$$L_0=\frac{2q_0}{\alpha}\frac{\partial W}{\partial x_0}\quad\text{where } q_0\triangleq\frac{\rho_\infty U^2}{2(D_{11})^{1/4}},\qquad M\triangleq\frac{U}{c_\infty}$$

$$\alpha\triangleq(M^2-1)^{1/2} \tag{3}$$

Hence, q_0 is the dynamic pressure in the affine plane. Equations (2) and (3), therefore, transform Eq. (1) to

$$\frac{\partial^4 W}{\partial x_0^4}+2D^*\frac{\partial^4 W}{\partial x_0^2\partial y_0^2}+\frac{\partial^4 W}{\partial y_0^4}+\rho h\frac{\partial^2 W}{\partial t^2}+\frac{2q_0}{\alpha}\frac{\partial W}{\partial x_0}$$

$$+N_{x_0}\frac{\partial^2 W}{\partial x_0^2}+N_{y_0}\frac{\partial^2 W}{\partial y_0^2}=0 \tag{4}$$

The solution to Eq. (4) for the simply supported panel can be written as

$$W=\mathrm{Re}[W_n(x_0)\sin(n\pi y_0/b_0)e^{i\omega t}] \tag{5}$$

where ω is the frequency (which, in general, is complex but since harmonic motion is considered, only the real value will be accepted here).

When Eq. (5) is substituted into Eq. (4) with the corresponding boundary conditions and then nondimensionalized (in several steps) the following equation is obtained:

$$\frac{\partial^4 W_n}{\partial\xi^4}+A\pi^2\frac{\partial^2 W_n}{\partial\xi^2}+\lambda\frac{\partial W_n}{\partial\xi}-kW_n=0 \tag{6}$$

where,

$$\xi\triangleq\frac{x_0}{a_0},\qquad A\triangleq(k_0-2D^*n^2)\left(\frac{a_0}{b_0}\right)^2,\qquad k_0\triangleq\frac{N_{x_0}b_0^2}{\pi^2}$$

$$\lambda\triangleq\frac{2q_0a_0^3}{\alpha},\qquad k\triangleq\rho h a_0^4\omega^2-\left(\frac{n\pi a_0}{b_0}\right)^4+\left(\frac{na_0}{b_0}\right)^2\pi^4 k_{y_0}$$

$$k_{y_0}=\frac{N_{y_0}a_0^2}{\pi^2} \tag{7}$$

Exact (Closed Form) Solution

To solve Eq. (6), the roots of its operator equation are written in the following form.

$$p_{1,2} = -\sigma \pm \beta, \qquad p_{3,4} = \sigma \pm i\delta \tag{8a}$$

Hence, the solution to Eq. (6) is of the form

$$W_n = A_1 e^{p_1\xi} + A_2 e^{p_2\xi} + A_3 e^{p_3\xi} + A_4 e^{p_4\xi} \tag{8b}$$

By substituting Eq. (8b) into Eq. (6) and applying the boundary conditions, the following frequency equation is obtained for a nontrivial solution.

$$8\sigma^2\beta\delta(\cosh 2\sigma - \cosh\beta\cos\delta) + \sin\delta\sinh\beta[(\beta^2+\delta^2)^2 + 4\sigma^2(\delta^2-\beta^2)] = 0 \tag{9a}$$

$$\beta = \left[\frac{\lambda}{4\sigma} - \sigma^2 - \frac{A\pi^2}{2}\right]^{1/2}, \qquad \delta = \left[\frac{\lambda}{4\sigma} + \sigma^2 + \frac{A\pi^2}{2}\right]^{1/2}$$

$$k = \frac{\lambda^2}{16\sigma^2} - 4\left[\sigma^2 + \frac{A\pi^2}{4}\right]^2 \tag{9b}$$

By specifying A and λ and using a root-solving subroutine the values of k are computed from Eqs. (9a) and (9b). Figures 2 and 3 display the results. For $\lambda = 0$, k corresponds to the natural frequencies for any specified k_0 and D^*. If $\lambda < \lambda_{cr}$, the frequencies are real, provided the particular panel is unbuckled, i.e., for any chosen D^*, k_0 should be less than the critical value (Ref. 19 has these critical values plotted). For $\lambda > \lambda_{cr}$, two values of k become complex which, in view of Eqs. (5) and (7), show that the panel must have at least one unstable mode of oscillation. Thus λ_{cr} defines the flutter boundary. In Fig. 3, the solid line shows the relationship between $(k_0 - 2n^2D^*)(a_0/b_0)^2$ and λ_{cr}. Notice how λ_{cr} decreases monotonically with $(k_0 - 2n^2D^*)(a_0/b_0)^2$; this means that for given values of k_0 and a_0/b_0, the lowest value of λ_{cr} would correspond to $n = 1$. Therefore, the shape of the flutter mode in the y_0 direction is a half-sine wave. It should also be noticed from Fig. 3 that the flutter speed is not a function of k_{y_0}. This is a favorable result, particularly for wing panels where the greatest stresses are in the spanwise (y_0) direction.

Galerkin Solution

To obtain a Galerkin solution of Eq. (6), consider the following mode shape.

$$W_n = \sum_{m=1}^{N} a_{mn}\sin(m\pi\xi) \tag{10}$$

When Eq. (10) is substituted into Eq. (6), then multiplied by $\sin r\pi\xi$, and integrated, the following sets of equations for the coefficients a_{mn} are obtained:

$$\left(m^4 - m^2A - \frac{k}{\pi^4}\right)a_{mn} - \frac{\lambda}{\pi^3}\sum_{n=1}^{N}\bar{L}_{mn,rn}a_{rn} = 0 \quad (m=1,2,\dots N) \tag{11}$$

where

$$\bar{L}_{mn,rs} = \frac{4}{\pi}[rm/(r^2-m^2)], \qquad n=s,\ m+r\ \text{odd} \tag{12}$$

Evaluating Eq. (11) for $N=2$, results in the following frequency equation for a nontrivial solution. [Although two mode solutions do not give the exact results, they are used for their simplicity and conservatism (see, also, Ref. 7).]

$$\lambda/\pi^4 = (3/8)[(k_0-2D^*)(a_0/b_0)^2 + k/\pi^4 - 1]^{1/2} \times [16 - 4(k_0-2D^*)(a_0/b_0)^2 - k/\pi^4]^{1/2} \tag{13}$$

The plot of λ vs k/π^4 from Eq. (13) is represented by the dashed line in Fig. 2.

To obtain the expression for the approximate λ_{cr} from Eq. (13), the equation $\partial\lambda/\partial k = 0$, must be satisfied. Hence,

$$\lambda_{cr}/\pi^4 = (9/16)[5 - (k_0-2D^*)(a_0/b_0)^2] \tag{14a}$$

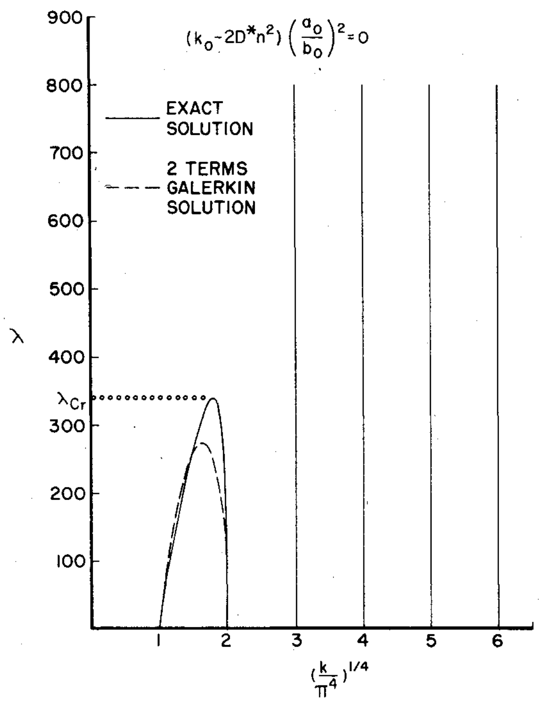

Fig. 2 Influence of airflow on orthotropic panel frequencies.

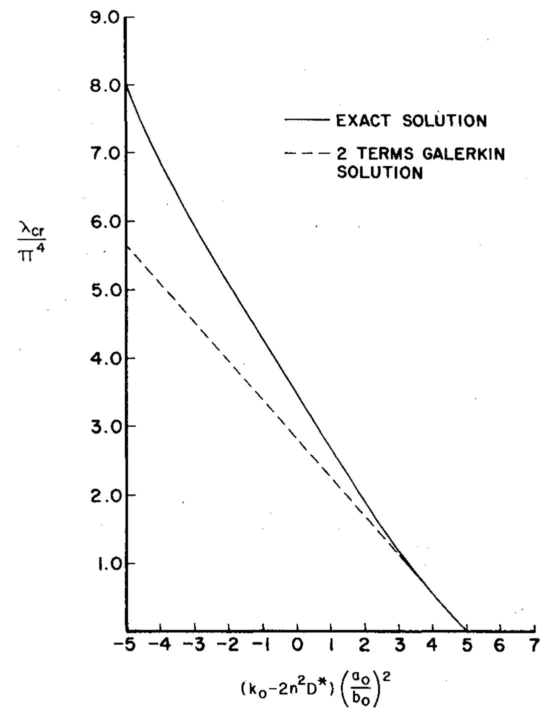

Fig. 3 Variation of λ_{cr} with $(k_0 - 2D^*n^2)(a_0/b_0)^2$ for two-dimensional orthotropic panel (strip theory).

Notice that λ_{cr} is independent of k_{y_0} and k, in agreement with the exact solution.

Similarly k_{cr} (obtained by satisfying $\partial k/\partial\lambda = 0$) is given by

$$k_{cr}/\pi^4 = (1/2)\,[17-5(k_0-2D^*)(a_0/b_0)^2] \tag{14b}$$

Aerodynamic Surface Theory

To check the accuracy of the aerodynamic strip theory which has been used in the previous section, the flutter of an unbuckled panel is being carried out in this section using the three-dimensional aerodynamic theory and Galerkin's method.

Aerodynamic Lift

Using source-superposition method, the lateral loading due to the aerodynamic forces can be estimated. That is, for a panel deflection of $W(x_0,y_0,t)$, the lateral load $L(x_0,y_0,t)$ is given by

$$L(x_0,y_0,t) = \rho U \frac{\partial\Phi}{\partial x_0}(x_0,y_0,t) \tag{15}$$

$$\Phi(x_0,y_0,t) = \frac{-U}{\pi}\iint \frac{\dfrac{\partial W}{\partial\tau}(\tau,\eta,t)\,\mathrm{d}\tau\mathrm{d}\eta}{[(x_0-\tau)^2-\alpha^2(y_0-\eta)^2]^{1/2}} \tag{16}$$

The integration being over the forward Mach cone from point (x_0,y_0).

Consider the mode shape given by

$$W = \mathrm{Re}\left[\sum_r\sum_s \sin(r\pi x_0/a_0)\sin(s\pi y_0/b_0)e^{i\omega t}\right] \tag{17}$$

over the region $0 \le x_0 \le a_0$, $0 \le y_0 \le b_0$. From Eq. (15),

$$L(x_0,y_0,t) = \mathrm{Re}\left[\sum_r\sum_s a_{rs}L_{rs}(x_0,y_0)e^{i\omega t}\right] \tag{18}$$

where

$$L_{rs} = 2q_0\frac{r}{a_0}\frac{\partial}{\partial x_0}\int_0^{x_0}\int_{y_0-1/\alpha(x_0-\tau)}^{y+1/\alpha(x_0-\tau)}\cos\left\{\frac{r\pi\tau}{a_0}\right\}\sin\left\{\frac{s\pi\eta}{b_0}\right\}$$
$$\times[\delta_0(\eta)-\delta_0(\eta-b_0)]\,\mathrm{d}\eta\mathrm{d}\tau \tag{19}$$

Modal Solution

When Eqs. (17-19) are substituted into Eq. (4) then multiplied by $\sin(m\pi x_0/a_0)\sin(n\pi y_0/b_0)$, and integrated the following set of equations is obtained:

$$\left[m^4-m^2(k_0-2n^2D^*)\left\{\frac{a_0}{b_0}\right\}^2-n^2\left\{\frac{a_0}{b_0}\right\}^2k_{y_0}\right.$$
$$\left.+n^4\left\{\frac{a_0}{b_0}\right\}^4-\omega_0^2\right]a_{mn}-\frac{\lambda}{\pi^3}\sum_r\sum_s\bar{L}_{mn,rs}a_{rs}=0 \tag{20}$$

where

$$\bar{L}_{mn,rs} = \frac{4\alpha mr}{a_0^2b_0}\int_0^{b_0}\int_0^{a_0}\int_0^{x_0}\int_{y_0-1/\alpha(x_0-\tau)}^{y_0+1/\alpha(x_0-\tau)}\cos\left\{\frac{m\pi x_0}{a_0}\right\}$$
$$\times\sin\left\{\frac{n\pi y_0}{b_0}\right\}\cos\left\{\frac{r\pi x_0}{a_0}\right\}\sin\left\{\frac{s\pi y_0}{b_0}\right\}L_I\,\mathrm{d}\eta\mathrm{d}\tau \tag{21}$$

where

$$L_I = \frac{[\delta_0(\eta)-\delta_0(\eta-b_0)]\,\mathrm{d}x_0\mathrm{d}y_0}{[(x_0-\tau)^2-\alpha^2(y_0-\eta)^2]^{1/2}}$$

Equation (21) gives the generalized forces, being functions of the modified aspect ratio parameter $(\alpha b_0/a_0)$. $\bar{L}_{mn,rs}$ is evaluated using an analysis similar to that shown in Appendix A of Ref. 2.

By restricting the analysis to Mach numbers greater than $(2)^{1/2}$, and keeping four modes in the x_0 direction and one mode in the y_0 direction, the flutter determinant obtained from Eq. (20) is solved using the computer. The plot is shown in Fig. 4. As is expected, λ_{cr} is independent of k_{y_0}. The result shows clearly that there is very little error in using the strip theory.

Buckled Panels

The previous sections show the profound effects of (k_0-2D^*) on flutter speed. Figure 5 shows [from Eqs. (8) and (13)] that for a given D^*, the pressure of the airstream tends to stabilize the panel, a fact that is generally assumed, based on the results of the isotropic panels.

In this section, the effects of buckling (resulting from compressive stresses) on flutter boundaries are analyzed. These stresses are generally caused by loading and aerodynamic heating, but for the purposes of this analysis, they are assumed to be caused entirely by aerodynamic heating. That is, a uniformly increased temperature ΔT is allowed to cause the buckling of the panel. Hence, N_{x_0} is given by†

$$N_{x_0} = h\beta_I\left[\alpha_0\Delta T-(1/2)\int_0^1\left\{\frac{\partial W}{\partial\xi}\right\}^2\mathrm{d}\xi\right] \tag{22}$$

where α_0 is the thermal coefficient and β_I a support factor which accounts for the effective stiffness of the supporting structure.

Equation (22) can be put in a slightly different form by defining a thermal stress coefficient k_0^T given by

$$k_0^T \triangleq \frac{h\beta_I\alpha_0\Delta T b_0^2}{\pi^2} \tag{23}$$

Hence,

$$N_{x_0} = \frac{k_0^T\pi^2}{b_0^2}-\frac{h\beta_I}{2}\int_0^1\left\{\frac{\partial W}{\partial\xi}\right\}^2\mathrm{d}\xi \tag{24}$$

By using Eq. (24), Eq. (4) can be modified slightly. Thus, in a nondimensionalized form, Eq. (4) becomes

$$\frac{\partial^4 W}{\partial\xi^4}+\left[(k_0^T-2n^2D^*)\pi^2\left\{\frac{a_0}{b_0}\right\}^2-\frac{a_0^2\beta_I h}{2}\int_0^1\left\{\frac{\partial W}{\partial\xi}\right\}^2\mathrm{d}\xi\right]$$
$$\times\frac{\partial^2 W}{\partial\xi^2}+\lambda\frac{\partial W}{\partial\xi}+\rho h a_0^4\frac{\partial^2 W}{\partial t^2}+[(n\pi a_0/b_0)^4$$
$$-n^2(a_0/b_0)^2\pi^4k_{y_0}]\,W=0 \tag{25}$$

By assuming $W=W_s+W_d$, where W_s and W_d represent static and dynamic deflections, respectively, and substituting into Eq. (25), the following two separate equations may be obtained.

†The cross-stream variation of N_{x_0} is ignored here to make the analysis easier. Strickly speaking, a postbuckling analysis should use large deformation theories (see Refs. 7 and 24).

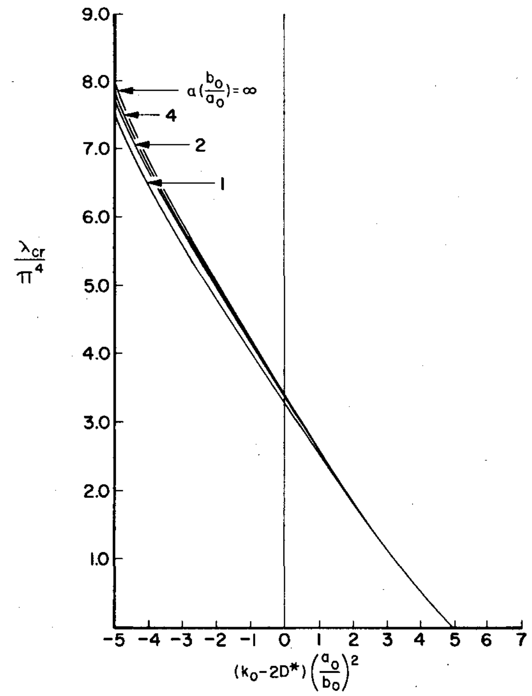

Fig. 4 Variation of λ_{cr} with $(k_0-2D^*)(a_0/b_0)^2$ for orthotropic panels (surface theory).

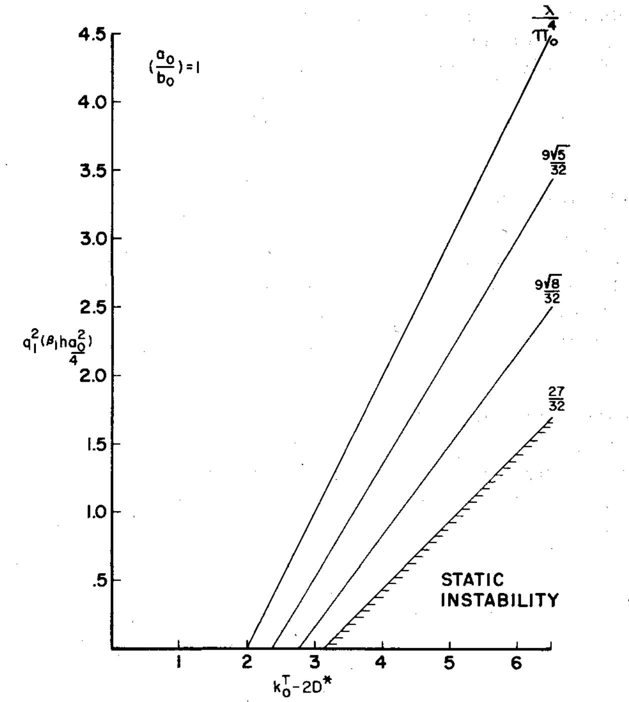

Fig. 6 Relations among first mode component, velocity, and temperature for buckled orthotropic panels.

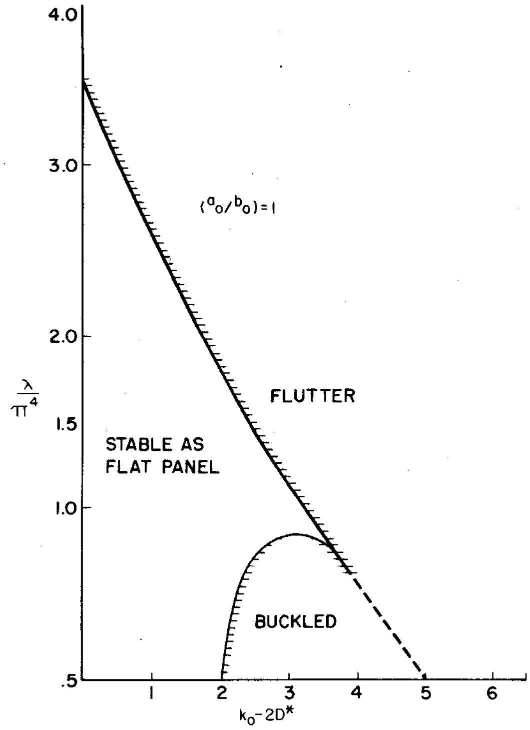

Fig. 5 Stability boundaries of two-dimensional flat orthotropic panels.

$$\frac{\partial^4 W_s}{\partial\xi^4} + \left[(k_0^T-2n^2D^*)\pi^2\left(\frac{a_0}{b_0}\right)^2 - \frac{a_0^2\beta_1 h}{2}\int_0^1\left\{\frac{\partial W_s}{\partial\xi}\right\}^2 d\xi\right] \times\frac{\partial^2 W_s}{\partial\xi^2} + \lambda\frac{\partial W_s}{\partial\xi} + \left[\left(n\pi\frac{a_0}{b_0}\right)^4 - n^2(a_0/b_0)^2\pi^4 k_{y0}\right]W_s = 0 \qquad (26)$$

$$\frac{\partial^4 W_d}{\partial\xi^4} + \left[(k_0^T-2n^2D^*)\pi^2\left(\frac{a_0}{b_0}\right)^2 - \frac{a_0^2\beta_1 h}{2} \times\int_0^1\left\{\frac{\partial W_s}{\partial\xi}+\frac{\partial W_d}{\partial\xi}\right\}^2 d\xi\right]\frac{\partial^2 W_d}{\partial\xi^2} + \lambda\frac{\partial W_d}{\partial\xi} - \frac{a_0^2\beta_1 h}{2} \times\left[\int_0^1 \frac{\partial W_d}{\partial\xi}\left\{2\frac{\partial W_s}{\partial\xi}+\frac{\partial W_d}{\partial\xi}\right\}d\xi\right]\frac{\partial^2 W_s}{\partial\xi^2} + \rho h a_0^4\frac{\partial^2 W_d}{\partial t^2} + \left[\left(n\pi\frac{a_0}{b_0}\right)^4 - n^2\left(\frac{a_0}{b_0}\right)^2\pi^4 k_{y0}\right]W_d = 0 \qquad (27)$$

Equations (26) and (27) may now be used to compute the static buckled deformation shape (W_s) and the additional dynamic deformation (W_d), respectively.

To solve Eq. (26) using Galerkin's method, consider the mode shape

$$W_s = \sum_{m=1}^{K} q_m \sin(m\pi\xi) \qquad (28)$$

By substituting Eq. (28) into Eq. (26), multiplying by $\sin(\ell\pi\xi)$, and integrating, a set of equations is obtained. When two modes are considered, the resulting set of equations (for $n=1$) are

$$\left[1-(k_0^T-2D^*+k_{y0})\left(\frac{a_0}{b_0}\right)^2+\left(\frac{a_0}{b_0}\right)^4 + \frac{a_0^2\beta_1 h}{4}(q_1^2+4q_2^2)\right]q_1 - \frac{8}{3}\frac{\lambda}{\pi^4}q_2 = 0$$

$$(8\lambda/3\pi^4)q_1 + [16-4(k_0^T-2D^*+\tfrac{1}{4}k_{y0})(a_0/b_0)^2 + (a_0/b_0)^4 + a_0^2\beta_1 h(q_1^2+4q_2^2)]q_2 = 0 \qquad (29)$$

The solution of Eqs. (29) leads to a quadratic equation in q_1 for any aspect ratio. However, in order to show the method of

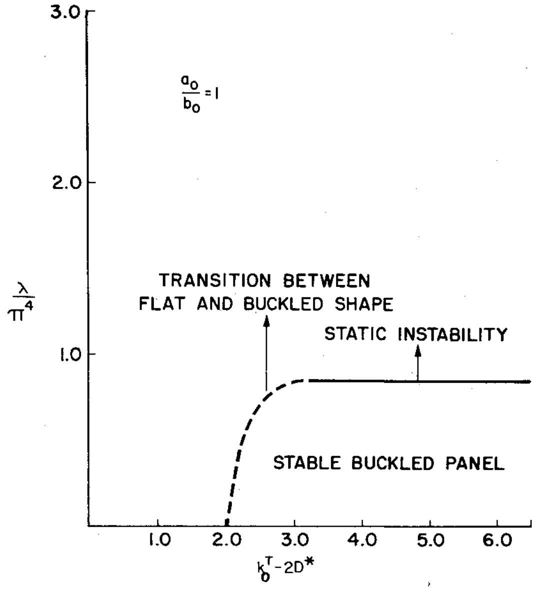

Fig. 7 Static stability boundaries of buckled orthotropic panels.

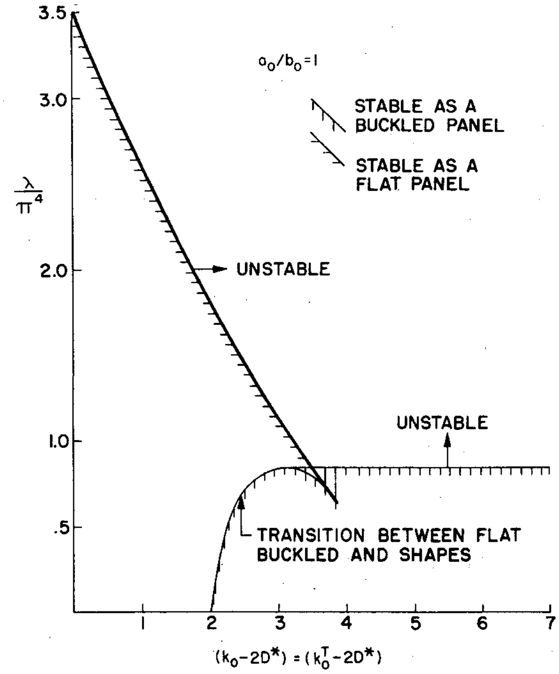

Fig. 8 Complete stability boundaries of flat and buckled orthotropic panels.

calculation, the following data is chosen; $a_0/b_0=1$, $k_{y_0}=0$. Consequently, by eliminating q_2 from Eqs. (29) q_1 becomes

$$q_1=\left[\frac{4}{a_0^2\beta_1 h}\left[\frac{1}{2}(k_0^T-2D^*-2)\left(1+\left\{1-\left(\frac{32\lambda}{27\pi^4}\right)^2\right\}^{1/2}\right)\times\left(\frac{8\lambda}{9\pi^4}\right)^2\right]\right]^{1/2} \tag{30}$$

The physics of this solution is portrayed by Fig. 6. It appears that for any given D^*, for $(k_0^T-2D^*)<2$, only the trivial solution, $q_1=q_2=0$ is obtained (i.e., $k_0^T<4$ for isotropic panels where $D^*=1$, or $k_0^T<2$ for $D^*=0$).[19] Hence, in Fig. 6 it can be observed that $q_1=0$ in the region where $(k_0^T-2D^*)<2$ (regions where loads are less than buckling loads). For the region where $2<(k_0^T-2D^*)<3.125$, it is observed that an increase in airspeed (or λ), will cause the buckle depth to decrease (q_1 decreases). But for $(k_0^T-2D^*)>3.125$, an increase in airspeed will cause the buckle depth to decrease until a static instability is reached. The velocity at which a buckled panel becomes statically unstable is independent of buckle depth and temperature. Figure 7 shows the plots of the boundaries of static instability.

In order to analyze the dynamic instability of buckled panels it is necessary to linearize Eq. (27) based on the philosophy that W_d represents a small perturbation in W and, hence, much smaller than W_s. Thus, Eq. (27) is linearized to give

$$\frac{\partial^4 W_d}{\partial\xi^4}+\left[(k_0^T-2D^*)\pi^2\left(\frac{a_0}{b_0}\right)^2-\frac{a_0^2\beta_1 h}{2}\int_0^1\left\{\frac{\partial W_s}{\partial\xi}\right\}^2\mathrm{d}\xi\right]\times\frac{\partial^2 W_d}{\partial\xi^2}+\lambda\frac{\partial W_d}{\partial\xi}-a_0^2\beta_1 h\left[\int_0^1\frac{\partial W_d}{\partial\xi}\frac{\partial W_s}{\partial\xi}\mathrm{d}\xi\right]\frac{\partial^2 W_s}{\partial\xi^2}+\rho h a_0^4\frac{\partial^2 W_d}{\partial t^2}+\left[\left(\pi\frac{a_0}{b_0}\right)^4-\left(\frac{a_0}{b_0}\right)^2\pi^4 k_{y_0}\right]W_d=0 \tag{31}$$

By specifying the mode shapes, Galerkin's method can be used to solve Eq. (31). Therefore, consider

$$W_d=\sum_{m=1}^{K}a_m\sin(m\pi\xi) \tag{32}$$

When Eqs. (28) and (32) are substituted into Eq. (31), and multiplied by $\sin(\ell\pi\xi)$, then integrated, the following equations are obtained for two modes.

$$\left[\frac{\rho h a_0^4}{\pi^4}\frac{\partial^2}{\partial s_0^2}+1-(k_0^T-2D^*+k_{y_0})\left(\frac{a_0}{b_0}\right)^2+\left(\frac{a_0}{b_0}\right)^4+a_0^2\beta_1 h\left(\frac{3}{4}q_1^2+q_2^2\right)\right]a_1+\left[2a_0^2\beta_1 h q_1 q_2-\frac{8}{3}\frac{\lambda}{\pi^4}\right]a_2=0 \tag{33a}$$

$$\left[2a_0^2\beta_1 h q_1 q_2+\frac{8}{3}\frac{\lambda}{\pi^4}\right]a_1+\left[\frac{\rho h a_0^4}{\pi^4}\frac{\partial^2}{\partial s_0^2}+16-4\left(k_0^T-2D^*+\frac{1}{4}k_{y_0}\right)\left(\frac{a_0}{b_0}\right)^2+\left(\frac{a_0}{b_0}\right)^4+a_0^2\beta_1 h(q_1^2+12q_2^2)\right]a_2=0 \tag{33b}$$

By assuming a_1 and a_2 to have solutions of the form, e^{us_0}, Eqs. (33) lead to a fourth-order characteristic equation in u. For $a_0/b_0=1$, and $k_{y_0}=0$, the values of q_1 and q_2 are obtained from Eqs. (29). The resulting characteristic equation is examined for static and dynamic stability by means of Routh's criteria. Thus, after the algebraic manipulation is carried out the characteristic equation takes the following form.

$$b_1u^4+b_2u^3+b_3u^2+b_4u+b_5=0 \tag{34}$$

From Routh's criteria, 1) there is static stability when all the coefficients are positive and 2) there is dynamic stability if $b_2b_3b_4-b_1b_4^2-b_5b_2^2>0$. When these conditions are investigated, the following conclusions are deduced. a) No dynamic instability is possible in the stable buckled range. b) For dynamic instability to occur for $(k_0^T-2D^*)<3.125$, the panel must be unbuckled. Consequently, the complete stability boundaries for orthotropic panels are obtained by combining Figs. 5 and 7, as shown in Fig. 8. [This represents the complete stability boundaries using the linearized theories (Ref. 24 discusses large deformation theories).]

Concluding Remarks

The affine transformations in this analysis have been known and used by people in the past. However, the importance and the range (bounds) of D^* were overlooked. Now it is established through the use of data and micromechanics that D^* varies between 0 and 1 for all orthotropic panels.[19] Hence, it is possible to present a unified panel flutter theory for isotropic and orthotropic panels, by making the analysis in this paper parallel to those previously presented for isotropic panels. Finally, it is hoped that any future analysis of orthotropic systems be done in the affine plane, since the physics seem to be easier to understand in this plane.

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